Measurements of Turbulent Flow Development in Tubes and Annuli with Square or Rounded Entrances

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Entrance-region studies, based on static-pressure measurements, were carried out with water flowing in annuli and a circular tube fitted with interchangeable square or rounded entrance sections. The tests covered the Reynolds number range from 16,000 to 70,000. For the annuli, the length of duct required to approach to within 5% of the fully developed pressure gradient was about 20 to 25 hydraulic diameters. This is in general accord with entrance length results for circular tubes and parallel-plate channels but differs from prior results for the annulus which had indicated entrance lengths larger by a factor of ten. The results for the sharp entrance showed a definite effect of separation and were characterized by high loss coefficients. For the rounded entrance, the initial part of the flow development was laminar; the entrance-region pressure drop did not substantially exceed (and in one case was less than) the corresponding fully developed pressure drop. Also for the rounded entrance, flow stability was improved and a monotonically decreasing pressure gradient obtained when a turbulent boundary layer was induced by means of a trip. Fully developed friction factors were calculated and compared with theory.

It is well known that a fluid passing into a duct from a chamber of different cross-sectional area undergoes a development of its velocity profile in the course of its flow through the duct. At a sufficient distance from the duct entrance, a balance between the pressure and viscous forces is achieved and an essentially fully developed, unchanging velocity profile results. That length of duct required to attain essentially fully developed conditions is designated as the *hydrodynamic entrance length.* For practical purposes, it is usually sufficient to associate the entrance length with the distance from the duct entrance that is needed to approach to within some specified percentage of the fully developed pressure gradient. Entrance lengths in turbulent flow are generally shorter than those in laminar flow. For turbulent flow in circular tubes or parallel-plate channels, the entrance length is usually on the order of 20 (hydraulic) diameters (*1*). Additionally, the turbulent entrance length is only moderately affected by the magnitude of the Reynolds number.

The only experiments relating to the hydrodynamic entrance region in annuli appear to be those of Rothfus and associates (*2*, *3*). The outer tube of their annulus had a square entrance, whereas their inner tube extended upstream into the supply reservoir. This geometry would probably be intermediate between the square and rounded entrances used in the present tests. Measurements are reported of the total shear force on the inner tube of various annuli (diameter ratios 2.97 and 1.78) with lengths of 4, 8, and 12 ft. The average skin friction on each 4-ft. section was assumed equal to the local values at the midpoint of the section. Results based on the ratio of the aforementioned local shear stress to the fully developed shear stress indicate an entrance region behavior which is strongly at variance with that for turbulent flow in tubes and parallel-plate channels. First of all, the entrance length inferred from the annulus data appears to be about an order of magnitude larger ($\sim$ 10 times) than the typical tube-channel entrance length. Second, the annulus entrance lengths appear to be strongly affected by the Reynolds number (over a range from 5,000 to 40,000 based on hydraulic diameter). It is not easy to understand why the annulus results should be so different from those of the tube and the channel, especially since a tube and a parallel-plate channel are the limiting cases of the annulus.

This investigation was initiated to explore the apparent anomaly discussed above. However, the scope was enlarged to include a number of other aspects of entrance-region flow. To achieve these aims, an experimental facility was constructed which consisted of a controlled water supply connected to a variety of test sections in which static-pressure measurements were made. Two annuli were employed: one with inner and outer diameters of 5/16 in. and 1 in. and the second with diameters of 1/2 in. and 1 in. These corresponded to diameter ratios of 3.2 and 2, respectively. Studies were also made in a circular tube of 1-in. diam. The test sections were designed so that tests could be carried out with both square and rounded entrances. The Reynolds number range of the experiments was from 16,000 to 70,000.

The aforementioned apparatus permitted detailed determination of the local pressure gradients in the entrance region as well as that for the fully developed region. With this information, entrance lengths could be determined. The effect of the entrance shape (square or rounded) on the flow pattern in the entrance region was investigated. The square entrance gave rise to a flow separation which was clearly evidenced by the measurements. The rounded entrance gave rise to a flow development in the duct which was initially laminar. Friction factors for fully developed flow in the annulus and the tube were calculated. The former were compared with the recent analyses of Deissler and Taylor (*4*) and of Meter and Bird (*5*). Pressure-cor-

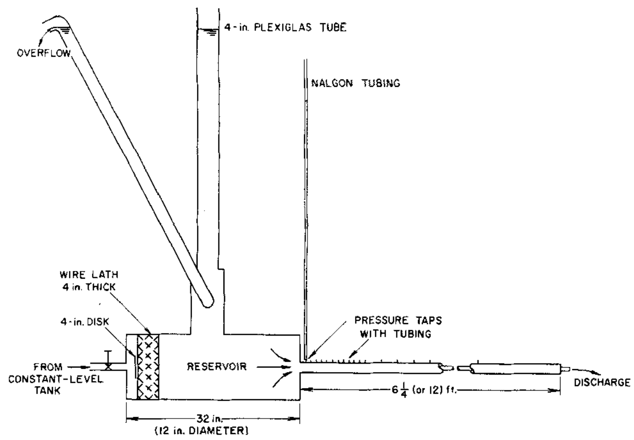

Fig. 1. Test apparatus.

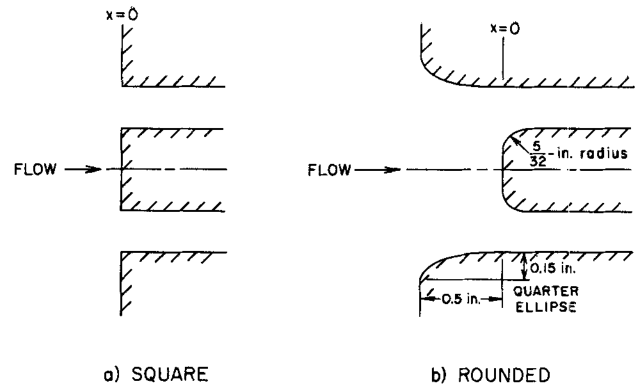

Fig. 2. Entrance forms.

rection terms and loss coefficients which take account of the entrance-region effects were determined and discussed for both the square and the rounded entrances.

EXPERIMENTAL APPARATUS

A schematic diagram of the flow system is presented in Figure 1. Water was supplied from a constant-level tank (not shown) located about 30 ft. above the test section. After it passed downward from the constant-level tank, the fluid entered a reservoir. The reservoir was fitted with a baffle plate and a screen section to calm the flow. The average velocity in the reservoir was on the order of 0.05 ft./sec. The flow rate was determined by collecting and weighing the discharge from the test section during a measured time interval. The maximum error in the flow-rate measurements is estimated to be $\pm$ 0.4% ($\pm$ 0.2 sec. in timing and $\pm$ ⅛ lb. in mass measurement). For nonweighing conditions, the discharge from the test section was collected in a sump and then returned to the constant-level tank. The water temperature was varied between 56.7°F. and 92.3°F. to extend the range of the Reynolds number.

A smooth brass tube with a 1-in. I.D. and ⅛-in. wall thickness was used as the outer tube of both of the annuli; it was also used in the tube-flow studies. The inner tubes used in the annulus tests had a ½-in. O. D. with 1/16-in. wall thickness and a 5/16-in. O.D. with 1/32-in. wall thickness, respectively. These various tubes were selected for straightness from dozens of available tubes.

The initial experiments were carried out in the ½ $\times$ 1-in. annulus and in the tube for a test-section length of 12 ft. This corresponded, respectively, to lengths of 288 hydraulic diameters for the annulus and of 144 diameters for the tube. This rather extended length was chosen so that entrance lengths as great as those of Rothfus could be investigated if necessary. The data from these tests indicated that developed flow was achieved in a relatively short length, and all subsequent experiments in both annuli and in the tube were made with a test-section length of 6.25 ft. For the tests in the annulus of 12-ft. length, the inner tube was supported by four pairs of struts located at intervals along the length. For the tests in the annuli of 6.25-ft. length, two pairs of struts were employed, the first pair being 13 in. from the entrance section and the second pair 14 in. from the exit section. The various inner tubes were positioned axially and centered at the downstream end with a ⅛-in. diametral locating pin. Holes for this pin were drilled only after careful positioning of the upstream ends of the tubes.

Extreme care was exercised in the fabrication of the static-pressure taps. These taps, located in the outer tube, were drilled on a boring mill and spaced to the nearest 0.001 in. Each tap was basically 1/32 in. in diam. but was enlarged to 1/16 in. diam. in the outer half of the ⅛-in. wall thickness. After the taps were drilled, the inner surface of the 1-in. tube was honed with a three-stone brake hone attached by a brass extension to a ¼-in. electric hand drill. Then, a 1/32-in. drill was twisted in each tap and the tube was rehoned throughout its entire length.

To facilitate pressure measurement, a short length of brass tube was soldered to a flat surface which had been machined with an end mill over each tap. Transparent plastic tubing (¼-in. I.D. Nalgon) was connected to each brass tube and suspended vertically above each tap location. The water surface in each of the plastic tubes had a flat meniscus, and preliminary tests showed no hysteresis effects for a rising or falling water column. Various other tubing which were investigated produced a capillary rise or depression when lowered into a free water surface; some showed hysteresis effects on the order of ¼ in. between levels for a rising vs. a falling liquid column.

Water levels in the just-described plastic piezometer tubes and in the reservoir were measured with the aid of a cathetometer. The readings were taken to the nearest 0.01 cm. of water. In the 12-ft. long test section, pressure taps were located every 3 in. for the first foot of length and at every foot or two thereafter. In the 6.25-ft. test section, there were taps at x = 1-3/16 in., 1-15/16 in., and every inch for the first foot, every 3 in. for the second foot and every foot thereafter.

To facilitate the study of the effect of entrance shape, the upstream ends of the inner tubes were provided with interchangeable nose plugs (Figure 2). One of these was square and the second was rounded (5/32-in. radius). Interchangeable square and rounded entrances were also provided for the 1-in. outer tube. The rounded entrance was made of clear plastic with a quarter ellipse. This ellipse was chosen to approximate a free streamline, although it was shortened to minimize boundary-layer thickness at the tube entrance.

Each set of struts supporting and positioning the inner tubes consisted of four 1/16-in. brass rods. These were the smallest brass rods available, and it was considered better to use them than smaller steel struts to preclude scratching the 1-in. tube during assembly. The struts were positioned at $\pm$ 45 deg. from the pressure tap locations. Test results later suggested that the struts increased the pressure gradient for at least 150 strut diameters downstream. That the struts caused this effect was verified in a final test for the 5/16 $\times$ 1-in. annulus, in which each of the upstream struts was filed to an aerodynamic shape.

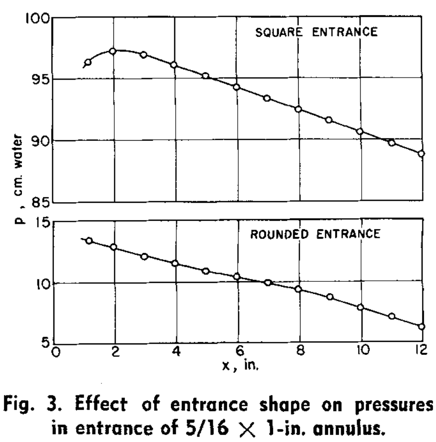

Fig. 3. Effect of entrance shape on pressures in entrance of 5/16 $\times$ 1-in. annulus.

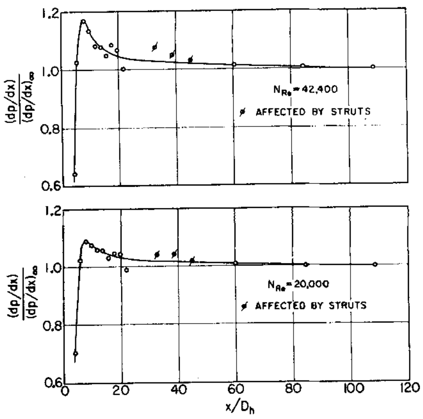

Fig. 4. Local pressure gradients for 1/2 × 1-in. annulus with a square entrance.

RESULTS

To illustrate the nature of the measurements, typical distributions of static pressure in the initial portion of the test section are presented in Figure 3 for the cases of the square entrance and the rounded entrance. These data correspond to the 5/16 × 1-in. annulus but are also representative of those for the other ducts. It is especially interesting to note the clear differences in the character of the pressure distribution as a function of the entrance shape (see also Figure 8). For the square entrance, the pressure decreases following the initial separation (this is not shown) then increases, and then decreases. The increase in pressure is related to the expansion of the stream which follows the vena contracta downstream of a sharp entrance. The length of the region of pressure recovery was shortest for the 1/2 × 1-in. annulus, slightly longer for the 5/16 × 1-in. annulus, and longest for the tube. For the rounded entrance, the pressure decreased monotonically with length. Closer inspection of this pressure curve reveals an inflection point which suggests a transition from laminar to turbulent flow.

Pressure gradients were computed from the original cathetometer readings and have been plotted in Figures 4 through 7. Central differences were used in the upstream section of the ducts, and downstream, where the pressure taps were farther apart, pressures between successive taps were differenced and plotted at the midpoint. Figures 4 and 5 respectively correspond to the 1/2 × 1-in. and the 5/16 × 1-in. annuli, both with square entrance. The pattern of point scattering near $x/D_h = 20$ in Figure 4 and $x/D_h = 15$ in Figure 5 is probably due to small local irregularities in pressure tap geometry and represents a

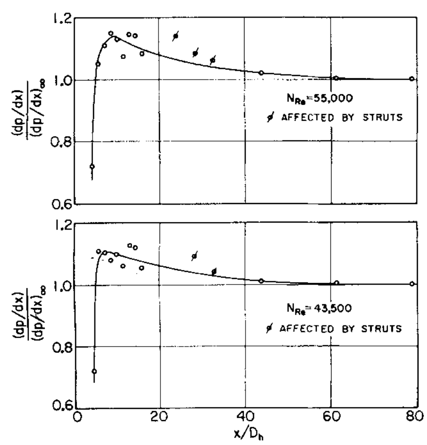

Fig. 5. Local pressure gradients for 5/16 × 1-in. annulus with a square entrance.

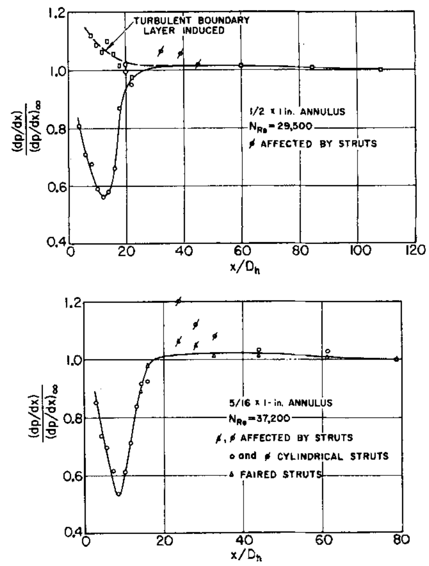

Fig. 6. Local pressure gradients for annuli with rounded entrances.

variation in measured pressure of less than ½% of the dynamic pressure of the mean flow ($0.005\ \rho V^2/2$) from the smoothed curve drawn. Figure 6 shows results for both annuli with rounded entrance. Figure 7 is for the tube and contains results for both square and rounded entrances. The ordinate represents the ratio of the local pressure gradient to the pressure gradient in the fully developed region. The abscissa is the distance from the entrance in terms of the hydraulic diameter.

An entrance length may be defined in terms of the length of duct needed for the local pressure gradient to approach to within 5% of the fully developed value. When Figures 4, 5, and 7 (upper part) are considered, it is seen that the entrance length for both annuli as well as for the tube is 25 hydraulic diameters or less for the case of the sharp entrance. A similar value applies for the tube with a rounded entrance, Figure 7 (lower part); however, this is reduced to less than 15 hydraulic diameters when a turbulent boundary layer is induced. For the annuli with rounded entrance, Figure 6 suggests that an entrance length of 20 hydraulic diameters is appropriate, with a reduction to about 15 hydraulic diameters with an induced turbulent boundary layer. The entrance lengths for the

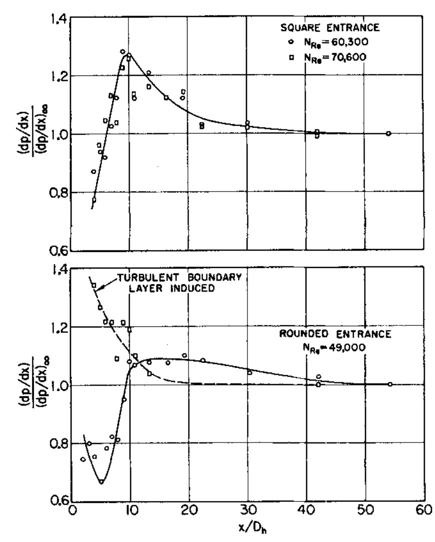

Fig. 7. Local pressure gradients for a smooth tube.

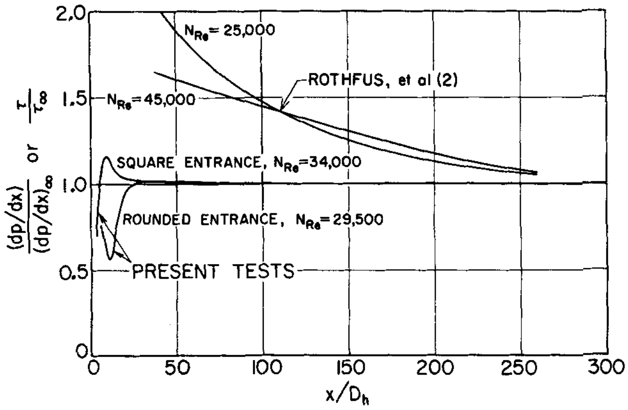

Fig. 8. Entrance length for smooth annuli based on wall shear reported by Rothfus et al. (2) compared with that based on pressure gradients from present tests.

annuli with rounded inlets should be affected by no more than one diameter because of the geometry of the inlet section (see Figure 2*b*).

These results are in general agreement with entrance lengths for turbulent flows in tubes and parallel-plate channels. However, the entrance lengths of Rothfus and associates for annuli are about an order of magnitude larger. A comparison of the present results with those of Rothfus is made in Figure 8. The latter are expressed as a ratio of local shear stress on the inner tube to the fully developed shear. The comparison is conservative, since the wall shear generally becomes constant first, then the pressure gradient, then the velocity profile. The shape of the entrance for the present tests had little effect; the entrance lengths were very nearly identical for the square and rounded entrances, at least relative to the abscissa scale of the data of reference 2 in Figure 8.

It should be noted that although the entrance lengths found by Rothfus and associates are about ten times those of this investigation, Figure 8 indicates that at the x/D_h value at which the flow is fully developed in the present tests, the shear measurements of Rothfus are only about 1.6 to 2 times their fully developed values.

It is of interest to highlight some of the details of the pressure gradient data of Figures 4 through 7. For the square entrance, Figures 4, 5, and 7 (upper part), the values of $(dp/dx)/(dp/dx)_\infty < 1$ at small x/D_h are due to the effects of pressure recovery downstream of the vena contracta. Due to this same phenomenon, the ratio of pressure gradients becomes negative at x values nearer zero. These points were not plotted to preserve an ordinate scale of reasonable size.

For the rounded entrance, Figures 6 and 7 (lower part), the pressure gradient curves indicate an initial laminar boundary layer which undergoes a transition to turbulence at the minimum point of the curve. The Reynolds number at the transition point was in the range of 250,000 to 350,000 based on the x- coordinate. This is a reasonable transition range for boundary layers which originate from a rounded leading edge. Results which are qualitatively similar to those of Figures 6 and 7 have been reported by Eckert and Irvine (*6*) for turbulent flow of air in triangular and rectangular ducts with rounded entrances and by Mills (*7*) for local heat transfer coefficients for turbulent flow in a tube with a bell-mouth entrance.

There were fluctuations in the flow with the rounded entrances which were manifested as fluctuations of about $\pm$ 0.06 cm. in the height of the water columns in the tubes connected to each pressure tap. It seems plausible that this was due to a shifting of the location of the boundary-layer transition back and forth near the duct entrance. Transition wandering has been observed in external flow in the boundary layer along a plate by Dryden (*8*).

It was believed that if a turbulent boundary layer were initiated near the duct entrance by means of a trip, a more stable flow would result with a monotonically decreasing pressure gradient similar to that postulated by the analysis of Deissler (*1*). A thin circular band 0.013 in. thick and ⅛ in. wide was placed in contact with the inner surface of the 1-in. outer tube, and a thin wire was wrapped around the inner tube of the ½-in. annulus. Flow stability was improved. There was a two-thirds reduction in the amplitude of the fluctuations of the piezometer water columns for the tube, and a nearly complete absence of these fluctuations for the annulus. Pressure gradients for this condition of induced turbulent boundary layer are shown in Figure 7 (lower part) and Figure 6 (upper part) for the tube and annulus, respectively. The expected monotonically decreasing pressure gradients were obtained.

On each one of the graphs in Figures 4, 5, and 6, there are two or three data points which are designated in a special way. These include the influence of the wake generated by the support struts of the annulus and therefore do not exclusively represent the pressure gradient due to wall shear and to the development of the velocity profile. The significance of the aforementioned wake effect was investigated by reshaping the 1/16-in. diam. strut rods into 0.025 × 0.062-in. faired struts for the 5/16 × 1-in. annulus. Measured pressure gradients for this assembly are designated by triangular points in Figure 6 (lower part). By inspection of the figure, it is seen that the data points immediately downstream of the struts are now in much closer accord with data for the remainder of the duct. This finding lends support to the neglect of the strut-affected points when passing curves through the data. That the wake of the struts should have an effect on the pressure gradient for downstream distances of 150 strut diameters may at first thought seem surprising. In this connection, it may be noted that Schlichting (*9*) and Sparks and Hoelscher (*10*) have measured influences on both the velocity and the turbulent intensity at distances of 208 and 180-239 diameters behind a cylinder in unconfined flows.

Entrance-Region Pressure Drop Corrections

In calculating the pressure drop in ducts, it is convenient to utilize fully developed friction factors and to add a correction term to account for entrance effects:

$$\frac{p_s - p}{\rho V^2/2} = f\frac{L}{D_h} + K \tag{1}$$

in which p_s is the pressure in a reservoir upstream of the duct (where the velocity is essentially zero) and L is the distance from the duct entrance at which the pressure is p. The correction term K accounts for the acceleration of the fluid from rest and the development of the velocity profile, the incremental viscous dissipation in the entrance region relative to that in the fully developed region, and separation losses, if any. K is a function of position in the entrance region, but becomes a constant in the fully developed flow regime. The fully developed value of K represents the accumulation of all the entrance-region effects sustained by the fluid. Practically speaking, it is the fully developed value of K which is of greatest utility. As an alternative to K, the entrance loss coefficient k_L may be defined as

$$k_L = K - 1 \tag{2}$$

Values of K and k_L computed from the test measurements are listed in Table 1. The effect of the struts was

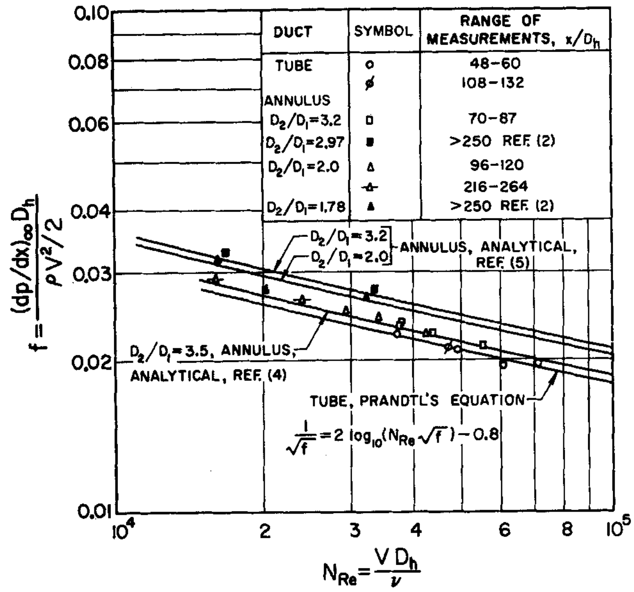

Fig. 9. Friction factors for smooth tubes and annuli.

calculated to be $\Delta p/(\rho V^2/2) = 0.12$ for the ½ × 1-in. annulus and 0.14 for the 5/16 × 1-in. annulus. These values agreed to within 5% with direct measurements of the pressure drop due to the struts. The K and k_L values reported below have been corrected for the presence of the struts.

The values of K (or k_L) for the square entrance are seen to be substantially larger than those for the rounded entrance without a trip, suggesting that losses due to separation are dominant. The k_L values of Table 1 agree well with published information in the hydraulic literature for square entrances.

The results for the rounded entrance are especially interesting. The k_L values are small or even negative without the trip. This is reasonable because the pressure gradients in both the initial laminar portion and the subsequent transition regime in the developing flow are smaller than those for the fully developed turbulent flow (Figure 6). In this instance, the entrance region does not necessarily increase the pressure drop relative to that for a fully developed flow. For a completely turbulent boundary layer obtained with a trip, the K and k_L values were higher than those without a trip. These higher values include the head loss of the trip itself and the higher frictional losses of a turbulent boundary layer relative to a laminar boundary layer. The size and shape of the trips were not optimized to obtain minimum loss coefficients.

Fully Developed Friction Factors

Pressure drop data from the fully developed regime were utilized in calculating friction factors according to the definition

TABLE 1. VALUES OF K AND k_L

Duct	Square entrance K	Square entrance k_L	Rounded entrance K	Rounded entrance k_L
1-in. tube	1.55	0.55	1.08	0.08
1-in. tube, with trip			1.14	0.14
5/16 × 1-in. annulus	1.54	0.54	1.02	0.02
1/2 × 1-in. annulus	1.52	0.52	0.98	−0.02
1/2 × 1-in. annulus, with trip			1.23	0.23

$$f = \frac{(dp/dx)_\infty D_h}{\rho V^2/2} \tag{3}$$

in which V is the mean velocity. These results are plotted in Figure 9. The circular tube results are compared with the Prandtl equation and good agreement is clearly evident. The results for the annuli are compared with the analytical predictions of Deissler and Taylor (*4*) and of Meter and Bird (*5*). The Deissler-Taylor curve corresponds to an annulus diameter ratio of 3.5 (the only concentric annulus treated by these authors). The present data generally fall between the predictions of the two analyses, with a tendency to favor that of Deissler and Taylor.

Typical annulus data of Rothfus et al. (*2*) for diameter ratios nearly the same as for the present tests lie slightly above the present data and are in general agreement with the results of the analysis of Meter and Bird.

The friction-factor results based on measurements in the region from $x = 4$ to 5 ft. are not distinguishably different from those based on measurements in the region from $x = 11$ to 12 ft. This tends to support the assertion that the present data are indeed fully developed.

ACKNOWLEDGMENT

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NOTATION

D_1 = inner tube diameter for annulus
D_2 = outer tube diameter for annulus
D_h = hydraulic diameter (equals D for a tube, and $D_2 - D_1$ for annulus)
f = friction factor $(dp/dx)_\infty D_h/(\rho V^2/2)$
K = coefficient in overall pressure drop [Equation (1)]
k_L = head loss coefficient [Equation (2)]
N_{Re} = Reynolds number (VD_h/ν)
p = pressure
p_s = total pressure in supply reservoir
V = average flow velocity in a duct
x = coordinate distance along duct axis (Figure 2)
ν = kinematic viscosity of fluid
ρ = fluid density

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